Modeling and Analysis of Coordinate System Transformation for a Dense Array Weapon System

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Abstract: Dense array system is the main short-range interception weapon at present, and its mathematical modeling research is of great significance. By establishing the ground coordinate system, the incoming aircraft coordinate system and the transitional coordinate system, and studying the mathematical relationship of the relative transformation among them, the coordinate matrix of the transformation is given, which provides a mathematical basis for the further study of the position relationship and hit judgment between the projectile and the aircraft.

1. Introduction

As the most famous shipborne short-range weapon system in the world, the "Dense Array" system is a short-range defensive naval gun weapon system, which is mainly used to deal with anti-ship aircrafts flying over the sea. The main features are fast reaction speed, high firing speed, high density of projectile curtain and full automation. It can automatically complete the whole process of target detection, threat judgment, tracking and locking, launching attack, damage assessment and fire control [1-3].

The remote control console of the "Dense Array" system is located in the bridge. Each console can control up to four groups of "Dense Array" systems, and can perform target assignment and monitoring. In addition, each "dense array" system has its own independent local console, which is usually located in the anti-seismic cabin near the "dense array" system to control the operation of the system. It can be used as a backup when the remote control console fails. The two consoles may also be used together. There are generally three operators: one shooter and two loaders [4-8].

The revolving machine gun manufactured by Green Company has six 76 times diameter and nine M16A1 barrels with right-handed rifles. It uses 20mm caliber ammunition (which can be replaced depending on the mission content) and has an effective range of 450-1800 meters (the maximum range record is 5486 meters). The rate of fire is about 3000-4500 rounds/min (depending on the type of ammunition). The original firing speed of this machine gun is 6600 rounds per minute, but it is slowed down to avoid the rapid depletion of ammunition. The base of the gun is MK-72 (originally called EX-83) of GD. The ammunition box at the bottom of the gun base can hold about 1,000 rounds of shells. In design, the "dense array" can intercept each target with about 200 rounds of shells. Therefore, theoretically, a "dense array" system filled with ammunition can engage five targets continuously [9-12]. However, the premise is that the five targets must appear in the same direction. If multiple targets attack from different directions at the same time, the "Dense Array" system needs to search again after the first battle, which wastes precious reaction time.The principle of action is to pour out a large amount of ammunition in a short time of firing, and form a very dense barrage on the path that the aircraft can pass through calculated by radar, so as to intercept and shoot down.

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2. Basic Hypothesis of Coordinate System Establishment

For the sake of simplicity, before deducing the hit judgment in the process of "dense array" firing simulation, the following assumptions are made generally:

Firstly, because the encounter time between aircraft and target ship is very short, the relative motion of aircraft and target can be considered as linear motion.

Secondly, the influence of the aircraft and the "dense array" projectile rolling around their respective central axes is not considered, because the "dense array" projectile is distributed along the circumference of space, so the hypothesis is reasonable.

Thirdly, the attitude angle of the anti-ship aircraft is neglected, that is to say, the velocity of the anti-ship aircraft coincides with its longitudinal axis, because the attitude angle of the "dense array" projectile is considered, and this hypothesis is valid according to the principle of relative motion.

Under the above assumptions, the coordinate system can be established and the transformation between coordinates can be realized.

3. Establishment of coordinate system

Ground coordinate system: $Oxyz$ is set as fig. 1, the origin is selected according to need; Ox axis is parallel to horizontal plane; Oy axis is perpendicular to horizontal plane, upward is positive; Oz axis is set form right-handed system. According to the mathematical model of the "dense array" projectile, the position and velocity of the projectile in the coordinate system can be easily obtained.

Aircraft body coordinate system: $O_m x_m y_m z_m$ is fixed with aircraft body, the origin in the center of the target anti-ship aircraft; $O_m x_m$ axis coincides with target longitudinal axis, and forward is positive (that is, pointing to head is positive); $O_m y_m$ axis is perpendicular to axis in the longitudinal symmetric plane of target, and upward is positive; $O_m z_m$ axis is perpendicular to plane $O_m x_m y_m$, and direction is determined according to right-handed rule. Anti-ship research is carried out in this coordinate system. The position coordinates of each part of the aircraft are very convenient.

Transitional coordinate system: $O_m x_0 y_0 z_0$ is fixed with aircraft body, the origin is also the center of anti-ship aircraft; $O_m x_0$ and $O_m y_0$ axes are parallel to and parallel to Ox and Oy in the ground coordinate system; $O_m z_0$ axes is set form right-handed system (that is, the origin of ground coordinate system is moved to the origin of aircraft body coordinate system).

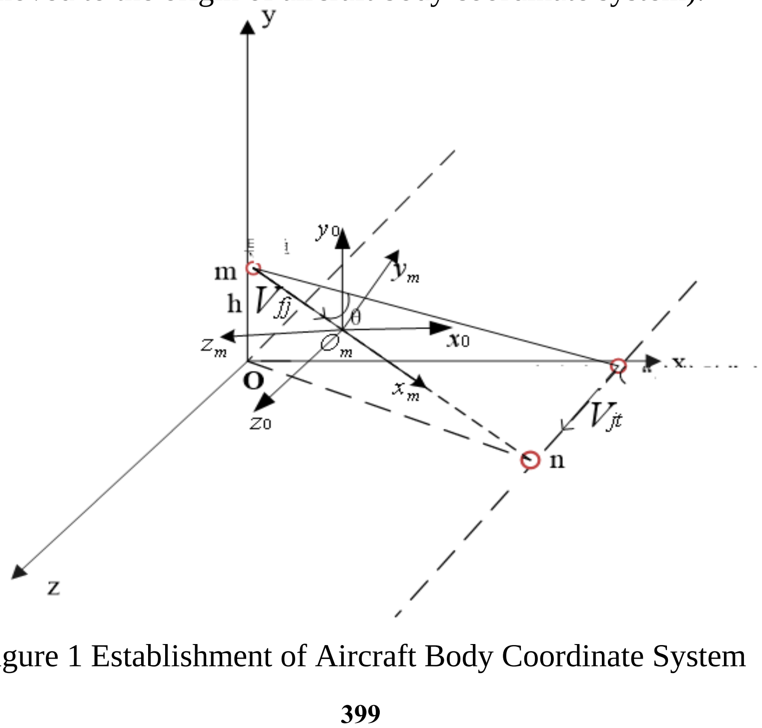

Figure 1 Establishment of Aircraft Body Coordinate System

4. Coordinate system transformation

After the aircraft body coordinate system is established, the position of any point on the aircraft can be conveniently expressed in the fixed aircraft body coordinate system. In addition, the space position and velocity of the "dense array" projectile in the ground coordinate system can also be obtained by the simulation model, and then coordinate transformation is carried out.

Directional cosine matrix is a very important matrix in coordinate transformation. It transforms the vector elements represented in one coordinate system into the vector elements represented in another coordinate system (usually accompanied by the rotation of the second coordinate system relative to the original coordinate system), and makes the vectors in the original coordinate system and in the second coordinate system. The vectors are quantitatively linked.

According to Figure 2, using simple trigonometry, the following relationships can be established:

$$y_1 = x_1 \cos\varphi + x_2 \sin\varphi \quad (1)$$

$$y_2 = x_1(-\sin\varphi) + x_2 \cos\varphi \quad (2)$$

The conversion matrix can be written as

$$\vec{C} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \quad (3)$$

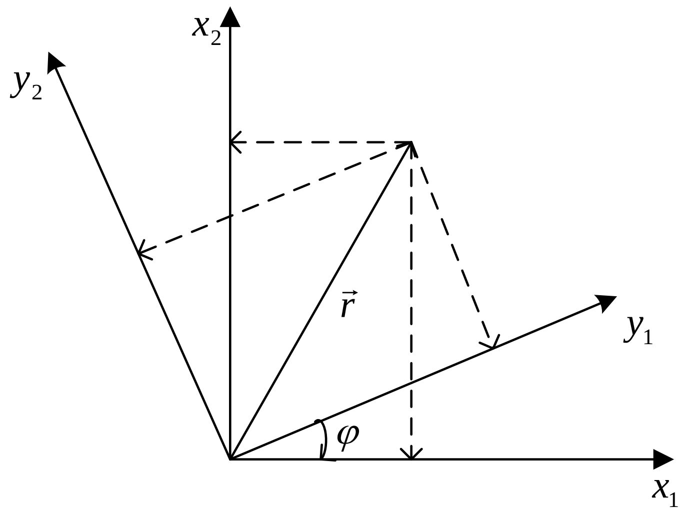

Figure 2 Component Relation Diagram of Vectors in Coordinate Systems

This matrix has several distinct features:

Firstly, the elements in the row (column) corresponding to the rotation axis in the direction cosine matrix are either 1 or 0.

Secondly, the other elements in the directional cosine matrix are either the sinusoidal value of the rotation angle or the cosine value of the rotation angle on the diagonal line, while the sinusoidal value is on the non-diagonal line.

Third, the negative sign in the sinusoidal term corresponds to the component outside the quadrant formed by the original coordinate system.

In summary, the relationship between aircraft body coordinate system and ground coordinate system can be obtained by one translation and three rotation of coordinate system.

So the transformation process is:

$$\begin{bmatrix} x_m, y_m, z_m \end{bmatrix}^T = \vec{L}(\gamma, \theta, \varphi) \begin{bmatrix} x_0, y_0, z_0 \end{bmatrix}^T \quad (4)$$

Where the rotation matrix can be found to be

$$\vec{L}(\gamma,\theta,\varphi)=\begin{bmatrix}\cos\theta\cos\varphi & \sin\theta & -\sin\varphi\cos\theta \\ -\sin\theta\cos\varphi\cos\gamma+\sin\varphi\sin\gamma & \cos\theta\cos\gamma & \sin\theta\sin\varphi\cos\gamma+\cos\varphi\sin\gamma \\ \sin\theta\cos\varphi\sin\gamma+\sin\varphi\cos\gamma & -\cos\theta\sin\gamma & -\sin\theta\sin\varphi\sin\gamma+\cos\varphi\cos\gamma\end{bmatrix} \quad (5)$$

5. Conclusion

In order to analyze the accurate spatial position relationship between projectile and aircraft in the dense array system and obtain the in-depth results of hit and damage, three coordinate systems, i.e. ground coordinate system, aircraft body coordinate system and transition coordinate system, are established in this paper, and the conversion relationship among them is studied, so as to further study. Dense array system has laid a good mathematical foundation.

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